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# Theoretical Calculation of Bending Stiffness of Alveolar Wall

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**Abstract** The bending stiffness of the alveolar wall is theoretically analyzed in this study through analytical modeling. First, the alveolar wall facet and its characteristics were geometrically simplified and then modeled using known physical laws. Bending stiffness is shown to be dependent on alveolar wall thickness, density, Poisson's ratio and speed of the longitudinal wave. The normal bending stiffness of the alveolar wall was further determined. For the adult human, the normal bending stiffness is calculated to be 71.0–414.7 nNm, while for the adult mouse it is 1.9–30.0 nNm. The results of this study can be used as a reference for future pulmonary emphysema and fibrosis studies, as the bending stiffness of alveolar wall will be lower and higher, respectively; than the theoretically determined normal values.

**Keywords** Stiffness · Theoretical modeling · Biomechanics · Pulmonary emphysema · Pulmonary fibrosis

## Introduction

The bending stiffness or flexural rigidity of the alveolar wall can act as a mechanical indicator of diseased or damaged lung parenchyma. Generally, the alveolar wall in lungs with pulmonary emphysema will have low stiffness values (Parameswaran et al. 2011), while the opposite is true for pulmonary fibrosis (Pellegrino et al. 2005).

Pulmonary emphysema is caused by alveolar tissue destruction, loss of small capillary blood vessels and digestion of elastin and/or collagen within the alveolar walls (de Ryk et al. 2007). Meanwhile, pulmonary fibrosis is caused by progressive replacement of normal parenchymal tissue with collagen-rich extracellular matrix, aberrant accumulation and activation of fibroblasts and remodeling of the alveolar wall tissue (Liu et al. 2010). Hence, pulmonary emphysema can be characterized by a reduction of stiffness, while pulmonary fibrosis can be characterized by an increase in stiffness.

Data concerning the normal bending stiffness of the alveolar wall for various animals are not available in the literature. The stiffness value of the alveolar wall is often dismissed as being too small and thus ignored in many experimental studies (Ma and Bates 2012).

In this study, the normal bending stiffness of the alveolar wall was theoretically analyzed and determined by modeling the alveolar wall facet as a flat membrane or plate. The results can be used as a reference for future experimental work and diagnosis concerning the condition of the lung parenchyma.

## Methods

### Modeling of Alveolar Structure

The component that defines the alveolar structure is its multifaceted alveolar wall. In this theoretical study, the facet of the alveolar wall along with its characteristics were simplified, modeled and analyzed. Some assumptions were made for the purpose of modeling.

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The alveolar wall contains elastin fibers and plasma membranes (with lipid bilayers). These elements furnish the alveolar wall with elastic properties. If the deformation experienced by the alveolar wall is considered reversible and smaller than its elastic limit, then the alveolar wall can be assumed to obey Hooke's law (Shabana 1995). The linear strain in the alveolar wall varies from 0 to 0.05 during normal tidal breathing (Roan and Waters 2011). Furthermore, Belete et al. (2010) state that wounding of cells in parenchymal tissue occurs significantly at a linear strain value of 0.08. Thus, this value is regarded here as the yield linear strain. Therefore, the linear theory of elasticity can be applied to the alveolar wall undergoing normal tidal breathing as the strain is below the elastic limit.

The alveolar wall is also assumed to be isotropic as it is homogenous (Gefen et al. 1999). The elasticity of the alveolar wall is characterized here with its elastic modulus, Poisson's ratio and strain components. The facet of the alveolar wall is shaped as a flat, irregular polygon (Prange 2003). Thus, the alveolar wall facet is assumed to be a flat membrane or plate.

### Bending Stiffness

The stiffness of membranes or plates is determined by the flexural rigidity, also known as bending stiffness. Bending stiffness is given as (Fung 1996)

$$D_e = Ed^3/12(1-u^2) \tag{1}$$

Bending stiffness depends on Young's modulus ($E$), thickness ($d$) and Poisson's ratio ($u$) of the membrane or plate.

### Elastic Modulus

An indirect calculation is conducted to obtain the equation for Young's modulus of the alveolar wall model. The speed of longitudinal sound waves ($c_L$) in biological soft tissue is dependent on the bulk modulus ($K$) and density ($\rho$) of the tissue (Matre and Dahl 2005):

$$c_L = (K/\rho)^{1/2} \tag{2}$$

The equation for Young's modulus ($E$) can be determined by utilizing its relationship with the bulk modulus. The Poisson's ratio of medium is denoted as $u$.

$$E = 3K(1-2u) \tag{3}$$

$$E = 3(\rho c_L^2)(1-2u) \tag{4}$$

### Bending Stiffness of the Alveolar Wall Model

The equation for bending stiffness of the alveolar wall model is obtained by substituting Eq. 4 into Eq. 1:

$$\begin{aligned} D_e &= Ed^3/12(1-u^2) \\ &= [3(\rho c_L^2)(1-2u)]d^3/12(1-u^2) \\ D_e &= 0.25d^3\rho c_L^2\frac{(1-2u)}{(1-u^2)} \end{aligned} \tag{5}$$

According to Eq. 5, the bending stiffness of the alveolar wall model depends on four quantities of the alveolar wall. Bending stiffness has power relationships with the thickness and the speed of the longitudinal wave but a negative relationship with Poisson's ratio. Bending stiffness further depends linearly on the density.

Information concerning the density, thickness, Poisson's ratio and speed of the longitudinal wave in pulmonary fibrosis and emphysema tissues is sparse. As gleaned from the literature, the density of emphysematous tissue is less (Guenard et al. 1992) but Poisson's ratio is estimated to be higher (Brewer et al. 2003) than in normal tissue. Meanwhile, in pulmonary fibrosis, the alveolar wall is thicker (Soto and Lucey 2009) and the speed of the longitudinal wave is slightly higher than in normal tissue (Boozari et al. 2010). Therefore, bending stiffness generally is smaller for pulmonary emphysema and higher for fibrosis compared to normal alveolar tissues.

## Results

### Normal Bending Stiffness

The alveolar wall is composed of soft tissues. Hence, the range of density of the alveolar wall in interval arithmetic notation is $\{1026, 1068\}$ kg/m$^3$, while the longitudinal wave propagation speed is $\{1490, 1610\}$ (Ludwig 1950). Furthermore, Poisson's ratio of the alveolar wall is $\{0.35, 0.45\}$ (Lai-Fook et al. 1976). Thus, substitution of these values into Eq. 5 results in

$$\begin{aligned} D_e &= 0.25d^3(\rho c_L^2)(1-2u)/(1-u^2) \\ &= 0.25d^3(\{1026,\ 1068\}\ )(\{1490,\ 1610\}^2) \\ &\quad (1-2[\{0.35, 0.45\}])/(1-[\{0.35, 0.45\}]^2) \\ D_e &= \{0.071, 0.24\}\ \times 10^9 d^3\ \text{Nm} \end{aligned} \tag{6}$$

The resulting bending stiffness curve of the alveolar wall model is depicted in Fig. 1. The normal bending stiffness of alveolar wall can be determined by applying the value of normal alveolar wall thickness into Eq. 6 or the Fig. 1.

### Verification

The modeling done in this study was verified through the comparison of calculated theoretical data (using Poisson's ratio = 0.35), with the experimental data of bending

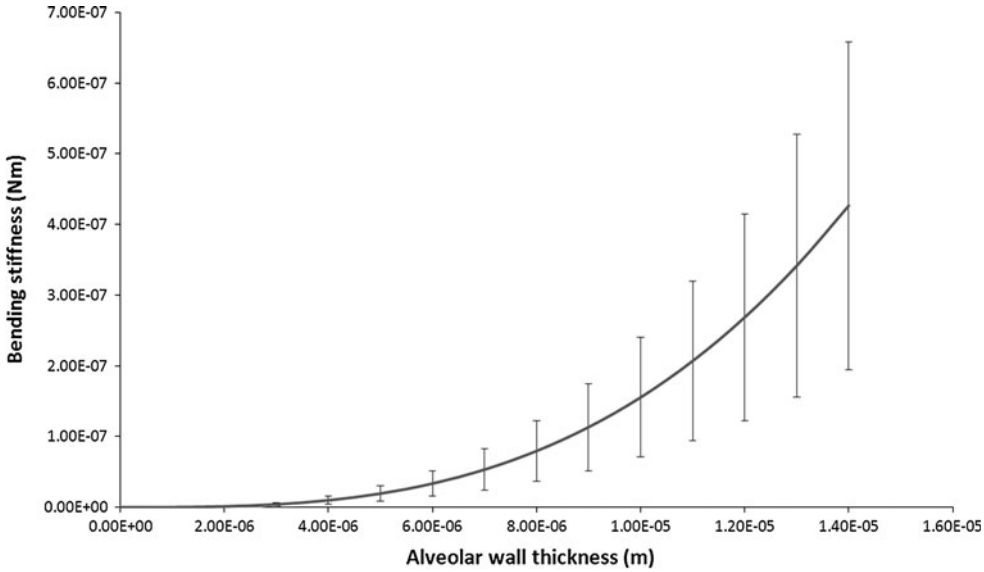


**Fig. 1** Ranges of bending stiffness of the alveolar wall model versus its thickness

**Table 1** Bending stiffness values of calculated theoretical data (using Poisson's ratio = 0.35) and of experimental data for egg-lecithin membrane (Bouvrais 2012) and polyethylene membrane (Sackmann 1995)

| Membrane type | Young's modulus (MPa) | Thickness (nm) | Bending stiffness (Nm) | |
|---|---|---|---|---|
| | | | Experimental | Calculated |
| Egg-lecithin | 28 | 3.6 | $2.0 \times 10^{-19}$ | $1.25 \times 10^{-19}$ |
| Polyethylene | 3,000 | 5.0 | $2.0 \times 10^{-17}$ | $3.60 \times 10^{-17}$ |

stiffness for egg-lecithin membrane (Bouvrais 2012) and for polyethylene membrane (Sackmann 1995) (Table 1). The calculated theoretical values are in line with the experimental data.

## Discussion

The range of values of normal bending stiffness depends primarily on the thickness of the alveolar wall, which varies with the type of species. For example, the adult mouse with a normal alveolar wall thickness of 3–5 μm (Choi et al. 2009) is predicted to have normal bending stiffness of the alveolar wall of 1.9–30.0 nNm. Meanwhile, for the adult human with a normal alveolar wall thickness of 10–12 μm (Choi et al. 2009), the range is 71.0–414.7 nNm. The normal bending stiffness curve of Fig. 1 can thus be utilized as a reference for normality in pulmonary emphysema and fibrosis studies of various species. The normal bending stiffness curve is also testable for future independent experimental validation of the modeling done here.

The lung parenchyma contains mostly collagens of Type I and Type III, which provide the structural framework and strength of the alveolar wall (Huang et al. 2007). The proteoglycans embed elastin fibers to the collagens, forming the extracellular matrix (ECM). The elastin fibers are thus mechanically connected to the collagens. The elastin-collagen network and the interaction of proteoglycans with elastin fibers provide elasticity in the alveolar wall (Cavalcante et al. 2005). There are also some interstitial smooth muscle cells in the ECM, which provide a viscoelastic property (Suki et al. 2005). The alveolar wall is built on the foundation of the ECM. The ECM acts like a hexagonal spring network (Ma and Bates 2012). A hexagonal spring matrix needs constraint, links and tethers to be stable and have a bending modulus (Krajcinovic 1996). Hence, the alveolar wall has a bending modulus (Khan et al. 2010) as it is interconnected with other alveolar walls, akin to the honeycomb structure. However, pulmonary emphysema and fibrosis can alter the constraint, links and tethers of the alveolar wall and, therefore, the bending rigidity of the alveolar wall.

In pulmonary fibrosis, the stiffness of the alveolar wall is increased and large pressure is needed for breathing, while it is the opposite for pulmonary emphysema. Several internal and external factors can alter the stiffness of the alveolar wall. The biochemical microenvironment and diseases may alter the ECM and cause emphysema or fibrosis (Suki et al. 2005). Furthermore, cigarette smoke increases the expression of collagen mRNA through elastase released by alveolar macrophages (Lucey et al. 1998). This scars the alveolar wall, resulting in fibrosis, which consequently increases the stiffness of the alveolar wall. Ventilator-induced lung injury and ultrasound-induced lung hemorrhage also can produce pulmonary fibrosis.

According to this study, the normal bending stiffness of the alveolar wall is small but not negligible. Hence, it should not be ignored in experimental studies while the

alveolar wall should be considered as a membrane instead a thin plate (Vepa 2010). This study is valid below the yield linear strain of the alveolar wall of 0.08 (Belete et al. 2010) as the alveolar wall was assumed to be linear.

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